

## 8.6 Solving Rational Equations

Example:  $\frac{3}{5x} = \frac{2}{x-7}$

$$3(x-7) = 10x$$

$$3x - 21 = 10x$$

$$\frac{-21}{7} = \frac{7x}{7}$$

$$x = -3$$

cross multiply

\* extraneous solutions

denom = 0!

Example:  $\frac{1}{2x+5} = \frac{x}{11x+8}$

$$11x+8 = 2x^2+5x$$

$$0 = 2x^2 - 6x - 8$$

$$0 = 2(x^2 - 3x - 4)$$

$$= 2(x-4)(x+1)$$

$x = 4, -1$

Example:  $\left(\frac{7}{2} + \frac{3}{x} = 3\right)$

$7x + 6 = 6x$

$x = -6$

Two methods to solve rational equations:

Cross-Multiply: use when each side has a single rational expression (proportion)

Use LCD- Solve by multiplying both sides by LCD to clear all fractions.

$$\text{Example: } \frac{3}{2} + \frac{4}{x-1} = \frac{x+1}{x-1}$$

$$3(x-1) + 8 = 2(x+1)$$

$$3x - 3 + 8 = 2x + 2$$

$$3x + 5 = 2x + 2$$

$$x = -3$$

Example:  $\frac{6}{x-3} = \frac{8x^2}{x^2-9} - \frac{4x}{x+3}$

$$\frac{6}{x-3} = \frac{8x^2}{(x-3)(x+3)} - \frac{4x}{x+3}$$

Handwritten annotations: Blue arrows point from the denominators  $(x-3)$  and  $(x+3)$  in the original equation to the corresponding terms in the partial fraction decomposition above. A blue circle highlights the  $(x-3)(x+3)$  denominator in the second term.

$$6(x+3) = 8x^2 - 4x(x-3)$$

$$6x+18 = 8x^2 - 4x^2 + 12x$$

$$6x+18 = 4x^2 + 12x$$

$$0 = 4x^2 - 6x - 18$$

$$0 = 2(2x^2 - 3x - 9)$$

$$0 = 2(2x+3)(x-3)$$

$$x = -\frac{3}{2} \quad \cancel{x=3} \text{ extraneous}$$

$$2x^2 - 6x + 3x - 9$$

$$2x(x-3) + 3(x-3)$$

Pg 593 5 – 8, 10, 11, 14 – 16, 19 – 21, 24,  
25

**CROSS MULTIPLYING** Solve the equation by cross multiplying. Check for extraneous solutions.

$$4. \frac{4}{2x} = \frac{5}{x+6}$$

$$5. \frac{9}{3x} = \frac{4}{x+2}$$

$$6. \frac{6}{x-1} = \frac{9}{x+1}$$

$$7. \frac{8}{3x-2} = \frac{2}{x-1}$$

$$8. \frac{x}{x+1} = \frac{3}{x+1}$$

$$9. \frac{x-3}{x+5} = \frac{x}{x+2}$$

$$10. \frac{x}{x^2-2} = \frac{-1}{x}$$

$$11. \frac{4(x-4)}{x^2+2x-8} = \frac{4}{x+4}$$

$$12. \frac{9}{x^2-6x+9} = \frac{3x}{x^2-3x}$$

**LEAST COMMON DENOMINATOR** Solve the equation by using the LCD. Check for extraneous solutions.

$$14. \frac{4}{x} + x = 5$$

$$15. \frac{2}{3x} + \frac{1}{6} = \frac{4}{3x}$$

$$16. \frac{5}{x} - 2 = \frac{2}{x+3}$$

$$17. \frac{1}{2x} + \frac{3}{x+7} = \frac{-1}{x}$$

$$18. \frac{1}{x-2} + 2 = \frac{3x}{x+2}$$

$$19. \frac{5}{x^2+x-6} = 2 + \frac{x-3}{x-2}$$

$$20. \frac{x+1}{x+6} + \frac{1}{x} = \frac{2x+1}{x+6}$$

$$21. \frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3}$$

$$22. \frac{6x}{x+4} + 4 = \frac{2x+2}{x-1}$$

$$23. \frac{10}{x} + 3 = \frac{x+9}{x-4}$$

$$24. \frac{18}{x^2-3x} - \frac{6}{x-3} = \frac{5}{x}$$

$$25. \frac{x+3}{x-3} + \frac{x}{x-5} = \frac{x+5}{x-5}$$