More 8-4

Identity-

An equation that is true for all values in the domain.

Examples: $\csc \theta = \frac{1}{\sin \theta}$

$$2(x + 3) = 2x + 6$$

$$\sin^2 x + \cos^2 x = 1$$

STRATEGIES FOR PROVING IDENTITIES

1. Know the fundamental identities.
2. Start with the more complicated member of the identity and transform it into the form of the simpler member.
3. When possible, express different functions in terms of the same function.
4. Simplify fractions:
   a. Transform complex fractions into simple fractions.
   b. Factor and Reduce.
5. DO NOT treat identities as equations. You can NOT do the same thing to both sides. You must prove they are equal, so we can't assume it is true.

Prove:

$\sin\theta \cdot \sec\theta \cdot \cot\theta = 1$

$\sin\theta \cdot \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}$

$\frac{\sin\theta \cdot \cos\theta}{\cos\theta \cdot \sin\theta}$

1

quotient/reciprocal

multiply

simplify

Prove:

$\cot\theta = \cos\theta \bullet \csc\theta$

$\cos\theta \cdot \frac{1}{\sin\theta}$ reciprocal

$\frac{\cos\theta}{\sin\theta}$ multiply

$\cot\theta$ quotient

Remember:

$$\sin^2\theta + \cos^2\theta = 1$$

We can derive 2 more "Pythagorean relationships".

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$* \quad \tan^2\theta + 1 = \sec^2\theta \quad *$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$* \quad 1 + \cot^2\theta = \csc^2\theta \quad *$$

Prove:

$$\frac{\cot A(1+\tan^2 A)}{\tan A} = \csc^2 A$$

$$\frac{\cot A(\sec^2 A)}{\tan A}$$

pythag.

$$\frac{\frac{\cos A}{\sin A}(\sec^2 A)}{\frac{\sin A}{\cos A}}$$

quotient

$$\frac{\cos^2 A}{\sin^2 A} \cdot \frac{1}{\cos^2 A}$$

flip/mult. reciprocal

$$\frac{1}{\sin^2 A}$$

mult. simp

$$\csc^2 A$$

recip.

$\csc^2 A$

Prove:

$$\frac{\cot x + \tan x}{\cot x} = \sec^2 x$$

$$\frac{\sin x}{\cos x}\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)$$

$$\frac{\cos x}{\sin x}$$

$$1 + \frac{\sin^2 x}{\cos^2 x}$$

$$1 + \tan^2 x$$

$$\sec^2 x$$