

8.1 Joint and Inverse Variation

Remember: Direct Variation

$$y = ax$$

a = constant of variation

What happens as x increases?
decreases?

Inverse Variation: $y = \frac{a}{x}$

$a =$ constant of variation

$a \neq 0$

What happens as x gets smaller?
larger?

Example: The variables x and y vary inversely. Write an equation that relates x and y if $y = 5$ when $x = 4$. Then find y when $x = -2$.

$$y = \frac{a}{x}$$

$$5 = \frac{a}{4}$$

$$a = 20$$

$$y = \frac{20}{x}$$

$$y = \frac{20}{-2}$$

$$y = -10$$

Example: Determine whether x and y show direct variation, inverse variation or neither.

A

X	Y
1	$5/2$
2	5
3	$15/2$
4	10

*

$$y = ax$$

$$y = \frac{a}{x}$$

if direct
 $5 = a(2)$
 $a = \frac{5}{2}$

$$y = \frac{5}{2}x$$

~~$$y = \frac{a}{x}$$

$$5 = \frac{a}{2}$$

$$a = 10$$~~

direct

B

X	Y
1	20
2	10
4	5
5	4

$$y = \frac{a}{x}$$

$$4 = \frac{a}{5}$$

$$a = 20$$

inverse

check to see if works.

$$y = \frac{20}{x}$$

Example: Determine whether x and y show direct variation, inverse variation or neither.

$$y = a x$$

$$y = \frac{a}{x}$$

a. $\frac{xy}{x} = \frac{5}{x}$

$$y = \frac{5}{x}$$

inverse

b. $y = x \oplus 5$

neither

c. $x = \frac{y}{5}$

$$y = 5x$$

direct

d. $y = \frac{x}{5}$

$$y = \frac{1}{5} x$$

direct

Joint Variation: a quantity varies directly with the product of two or more other quantities.

$$z = axy$$

a = constant of variation

Example: z varies jointly with x and y. $z = -150$ when $x = 3$ and $y = -5$. Write an equation to model this variation. Then find z when $x = 2$ and $y = 6$.

$$\begin{aligned} z &= axy \\ -150 &= a(3)(-5) \\ -150 &= -15a \\ a &= 10 \\ z &= 10xy \end{aligned}$$

$$\begin{aligned} z &= 10(2)(6) \\ z &= 120 \end{aligned}$$

Example: Write an equation for the given relationship.

a. x varies inversely with y and directly with w.

$$x = \frac{aw}{y}$$

$$\frac{a}{y} \cdot w$$

a = constant
of
variation

b. p varies jointly with q and r and inversely with the square of s.

$$p = \frac{aqr}{s^2}$$

$$\frac{a}{s^2} qr$$

a = constant of
variation

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DETERMINING VARIATION Tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

3. $xy = \frac{1}{5}$

4. $y = x + 4$

5. $\frac{y}{x} = 8$

7. $y = \frac{2}{x}$

8. $x + y = 6$

9. $8y = x$

USING INVERSE VARIATION The variables x and y vary inversely. Use the given values to write an equation relating x and y . Then find y when $x = 3$.

12. $x = 5, y = -4$

13. $x = 1, y = 9$

14. $x = -3, y = 8$

15. $x = 7, y = 2$

16. $x = \frac{3}{4}, y = 28$

17. $x = -4, y = -\frac{5}{4}$

18. $x = -12, y = -\frac{1}{6}$

19. $x = \frac{5}{3}, y = -7$

INTERPRETING DATA Determine whether x and y show *direct variation*, *inverse variation*, or *neither*.

20.

x	y
1.5	40
2.5	24
4	15
7.5	8
10	6

21.

x	y
12	132
18	198
23	253
29	319
34	374

22.

x	y
4	16
5	11
6.2	10
7	9
11	6

23.

x	y
4	21
6	14
8	10.5
8.4	10
12	7

USING JOINT VARIATION Write an equation relating x , y , and z given that z varies jointly with x and y . Then find z when $x = -4$ and $y = 5$.

24. $x = 2, y = -6, z = 24$

25. $x = 8, y = 6, z = 12$

26. $x = -\frac{1}{4}, y = -3, z = 15$

27. $x = 6, y = -7, z = -3$

28. $x = 9, y = -2, z = 6$

29. $x = 5, y = -3, z = 75$

WRITING EQUATIONS Write an equation for the given relationship.

31. x varies directly with y and inversely with z .

32. y varies jointly with x and the square of z .

33. w varies inversely with y and jointly with x and z .

37. **DIGITAL CAMERAS** The number n of photos your digital camera can store varies inversely with the average size s (in megapixels) of the photos. Your digital camera can store 54 photos when the average photo size is 1.92 megapixels. Write a model that gives n as a function of s . How many photos can your camera store when the average photo size is 3.87 megapixels?



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