6.4 Exploring Inverse Functions

MATERIALS • graph paper • straightedge

QUESTION How are a function and its *inverse* related?

EXPLORE Find the inverse of $f(x) = \frac{x - 3}{2}$

STEP 1 ***Graph function*** Choose values of x and find the corresponding values of $y = f(x)$. Plot the points and draw the line that passes through them.

STEP 2 ***Interchange coordinates*** Interchange the x- and y-coordinates of the ordered pairs found in Step 1. Plot the new points and draw the line that passes through them.

STEP 3 ***Write equation*** Write an equation of the line from Step 2. Call this function g.

STEP 4 ***Compare graphs*** Fold your graph paper so that the graphs of f and g coincide. How are the graphs geometrically related?

STEP 5 ***Describe functions*** In words, f is the function that subtracts 3 from x and then divides the result by 2. Describe the function g in words.

STEP 6 ***Find compositions*** Predict what the compositions $f(g(x))$ and $g(f(x))$ will be. Confirm your predictions by finding $f(g(x))$ and $g(f(x))$.

The functions f and g are called *inverses* of each other.

DRAW CONCLUSIONS Use your observations to complete these exercises

Complete Exercises 1–3 for each function below.

$f(x) = 3x + 2$ $\quad$ $f(x) = \frac{x - 1}{6}$ $\quad$ $f(x) = 4 - \frac{3}{2}x$

Inverse Relation: interchange the input and output values of the original relation.
(domain and range are switched)

If both the original and inverse are functions, called inverse functions.

KEY CONCEPT *For Your Notebook*

Inverse Functions

Functions f and g are inverses of each other provided:

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x$$

The function g is denoted by f^{-1}, read as "f inverse."

Example: Find the inverse of the given function. Verify that your result and the original function are inverses.

$f(x) = 2x + 1$

$x = 2y + 1$

$x - 1 = 2y$

$\boxed{\frac{x-1}{2} = y}$ $f^{-1}(x)$

inverse

Verify

$f(f^{-1}(x))$

$2\left(\frac{x-1}{2}\right)+1$

$(x-1)+1$

x

$f^{-1}(f(x))$

$\frac{(2x+1)-1}{2}$

$\frac{2x}{2}$

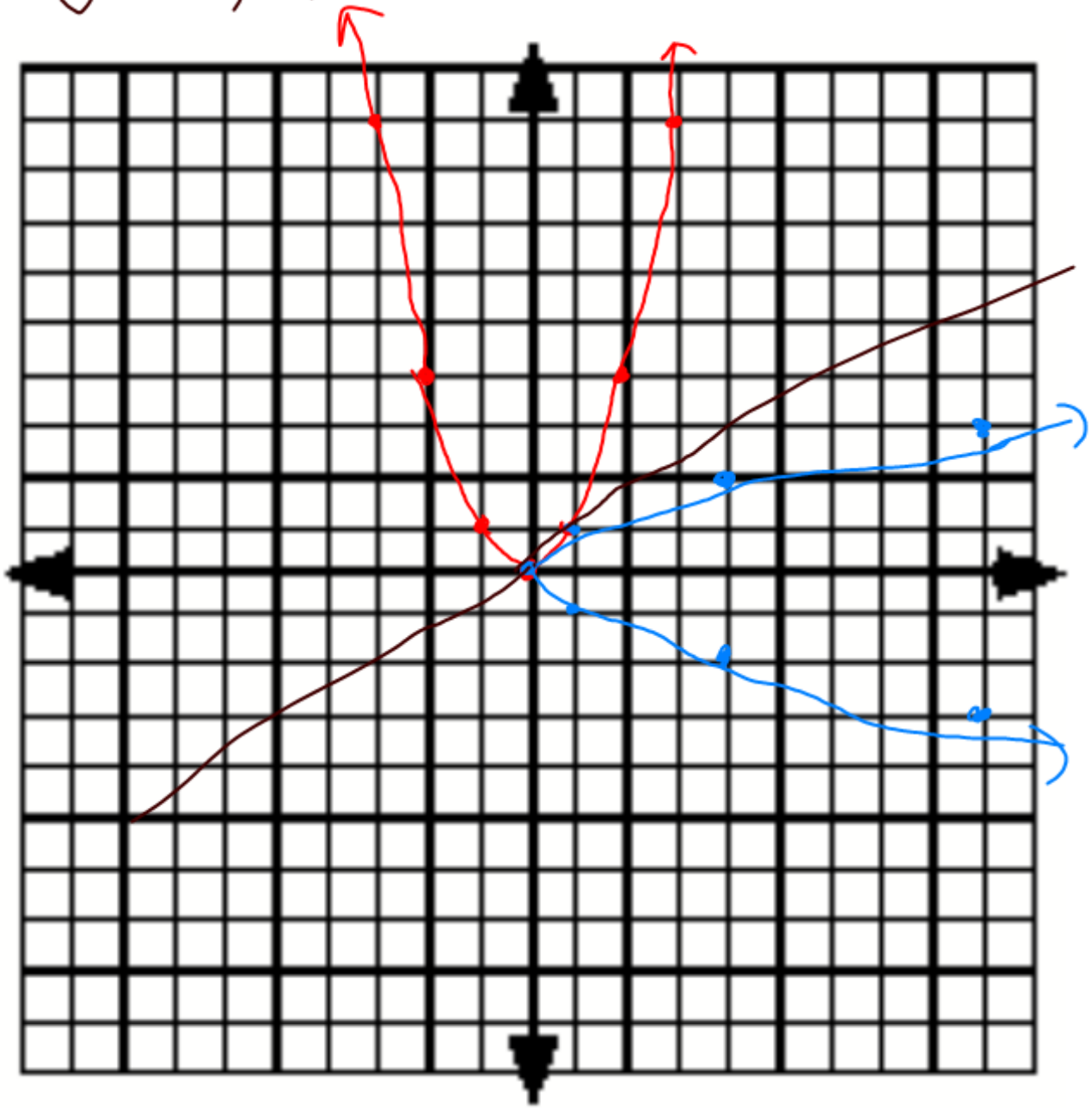
x

Graph $y = x^2$ and its inverse.

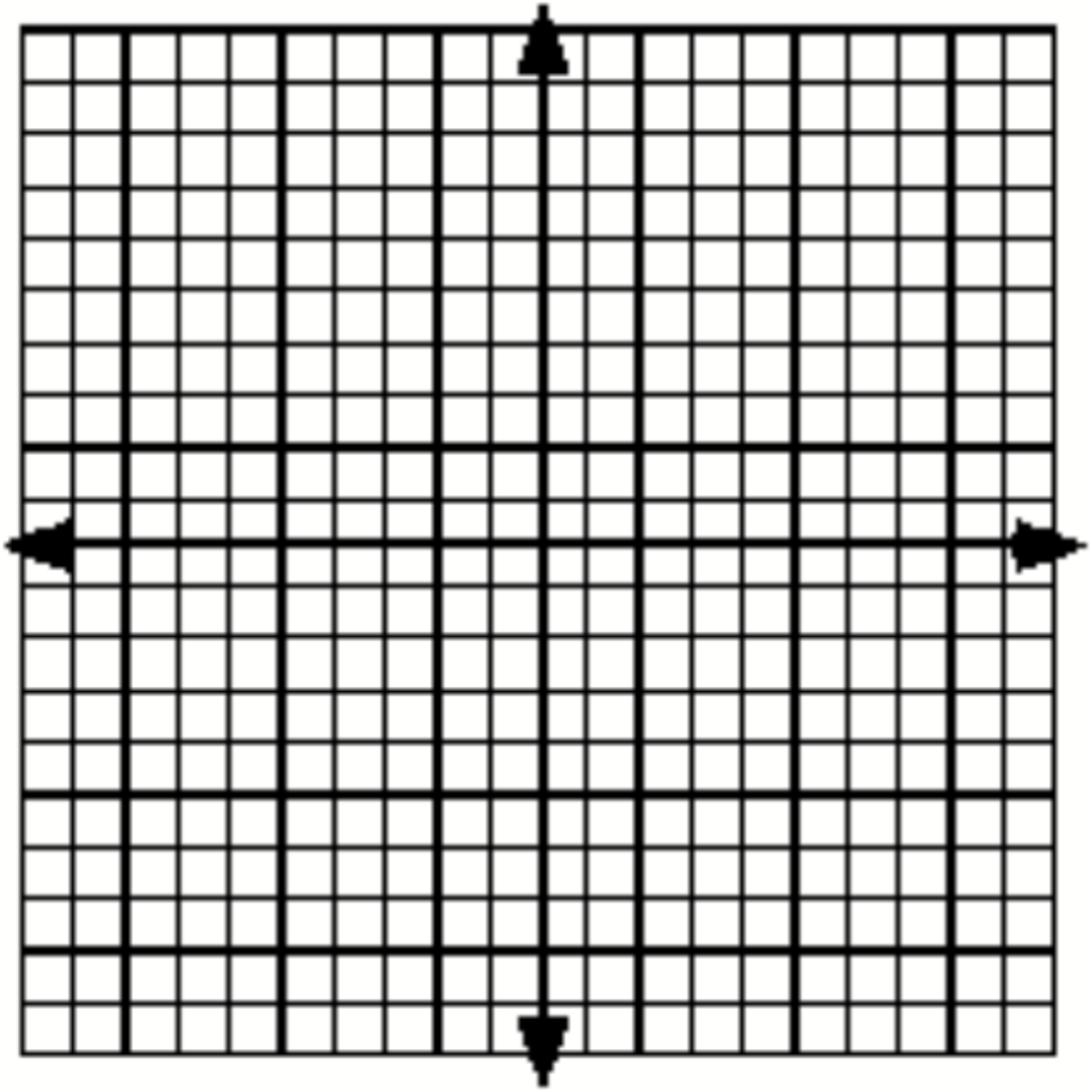
$y = x^2, \; x \geq 0$

(0,0) (1,1) (2,4) (3,9) (-1,1) (-2,4) (-3,9)

(0,0) (1,1) (4,2) (9,3) (1,-1) (4,-2) (9,-3)



Graph $y = x^3$ and its inverse.

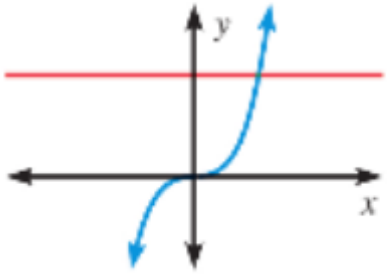


KEY CONCEPT *For Your Notebook*

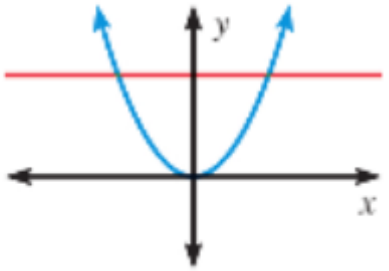
Horizontal Line Test

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.

Inverse is a function

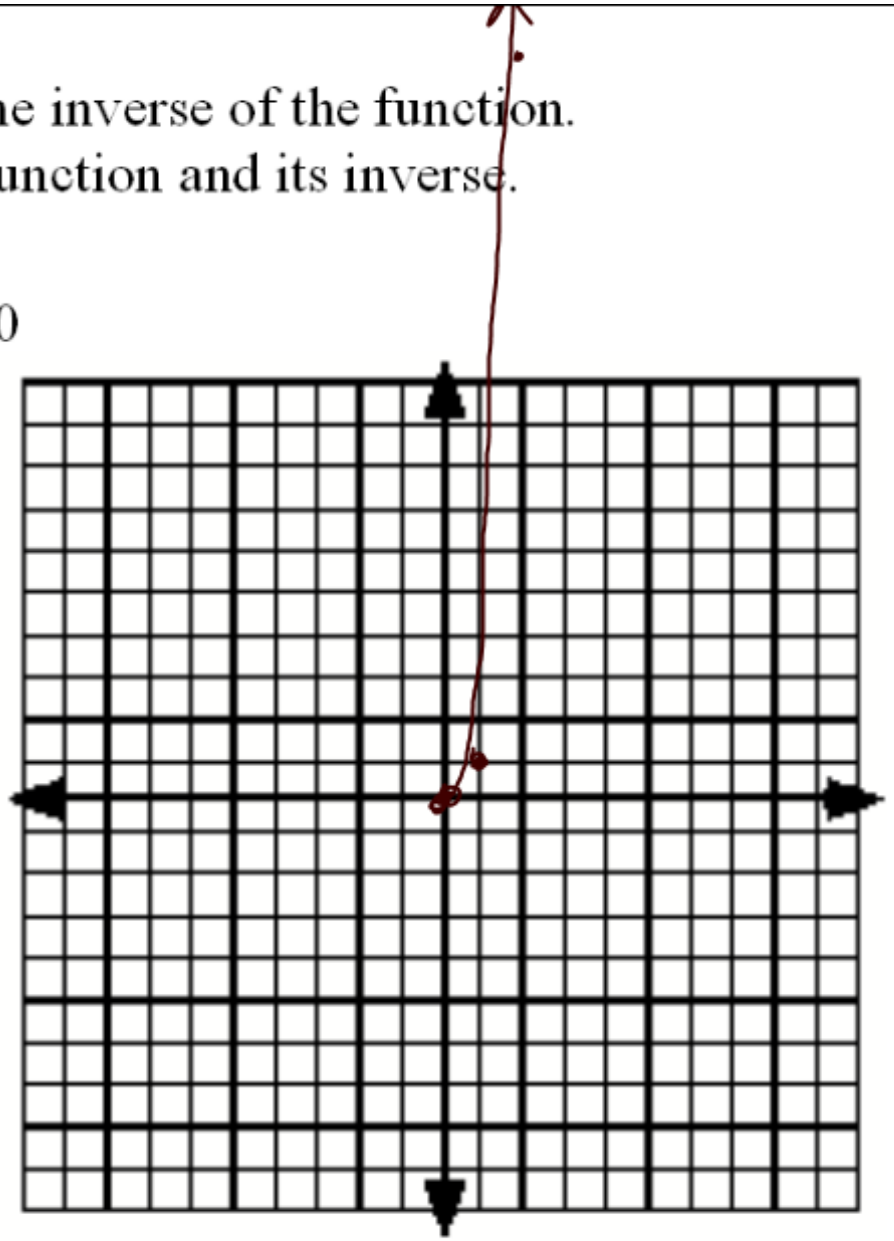


Inverse is not a function



Example: Find the inverse of the function.
Then graph the function and its inverse.

$F(x) = x^6,\ x \geq 0$



Example: Find the inverse of the function. Then graph the function and its inverse.

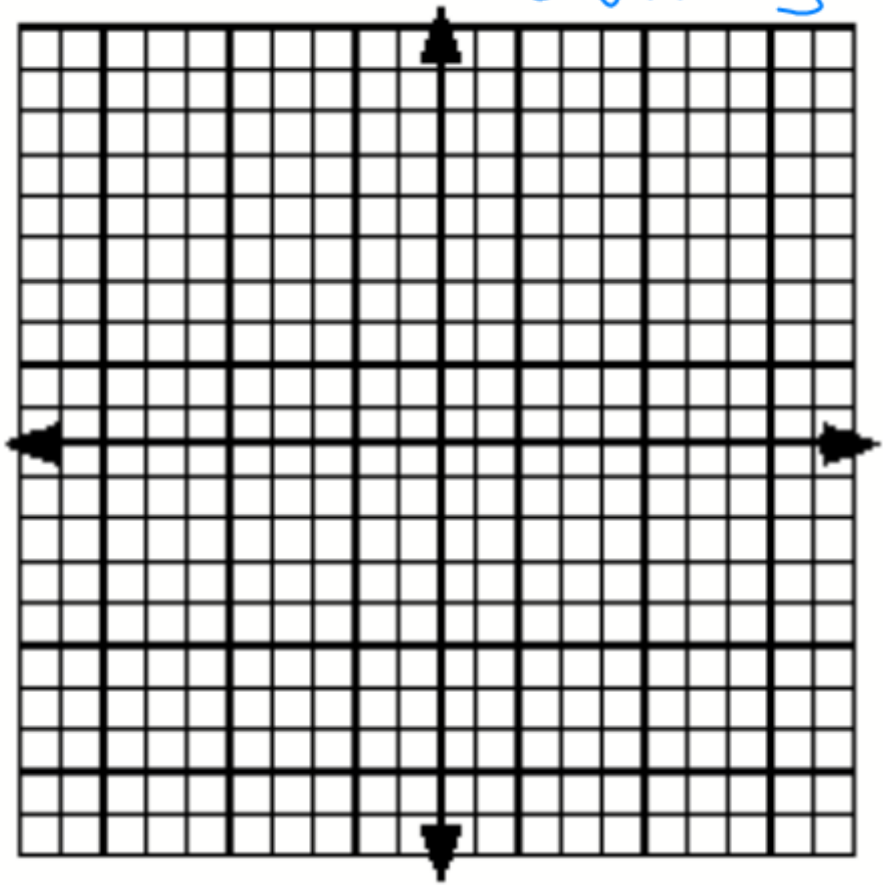
$$F(x) = \frac{1}{27}x^3$$

$y = \frac{1}{27}x^3$

$x = \frac{1}{27}y^3$

$\sqrt[3]{27x} = \sqrt[3]{y^3}$

$3\sqrt[3]{x} = y$



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Pg 442: Do on graph paper. Label graphs.
Simplify denominator.
~~4 – 10, 15, 17, 19~~, 22 – 25, 29, 31, 34 – 36, 39, 43