5-6
Laws of Logarithms

Inverses:

Find the inverse of $y = b^x$

$x = b^y$

$\log_b x = y$

inverses

Find the inverse of $y = \log_b x$

$x = \log_b y$

$b^x = y$

inverses

$\log_b MN = \log_b M + \log_b N$

Proof: let $\log_b M = x$ and $\log_b N = y$

$b^x = M$ $\quad b^y = N$

$b^x \cdot b^y = MN$

$b^{x+y} = MN$

$\log_b MN = x+y$

$\log_b MN = \log_b M + \log_b N$

Laws of Logarithms:

1. $\log_b MN = \log_b M + \log_b N$
2. $\log_b \frac{M}{N} = \log_b M - \log_b N$
3. $\log_b M = \log_b N$ if and only if $M = N$
4. $\log_b M^k = k \cdot \log_b M$

Examples:

1. Express as a single logarithm

a. $\log_5 2 + \log_5 3$ $\log_5(2 \cdot 3)$ $\log_5 6$

b. $\log 3 + \log 6 - \log 2$ $\log 18 - \log 2 = \log \frac{18}{2}$ $\log 9$

c. $1/2 \ln 25 - \ln 2$ $\ln 25^{1/2} - \ln 2$ $\ln 5 - \ln 2$ $\therefore \ln \frac{5}{2}$

2. Write in terms of log M and log N

a. $\log M^2 N$ $= \log M^2 + \log N$ $2 \log M + \log N$

b. $\log \sqrt{\frac{M}{N}}$ $\log \left(\frac{M}{N}\right)^{1/2}$ $\frac{1}{2} \log \frac{M}{N}$ $\frac{1}{2}(\log M - \log N)$.

3. If $\log_6 2 = x$ and $\log_6 5 = y$, express each logarithm in terms of x and y.

a. $\log_6 10$ $= \log_6 (2 \cdot 5)$

$\log_6 2 + \log_6 5$

$x + y$

b. $\log_6 40$ $= \log_6 (2 \cdot 2 \cdot 2 \cdot 5)$

$\log_6 2 + \log_6 2 + \log_6 2 + \log_6 5$

$x + x + x + y = 3x + y$

$\log_6 (2^3 \cdot 5)$

$3\log_6 2 + \log_6 5$

$3x + y$

c. $\log_6 3$ $= \log_6 \left(\frac{6}{2}\right) = \log_6 6 - \log_6 2$

$1 - x$

d. $\log_6 15$ $= \log_6 \left(\frac{6 \cdot 5}{2}\right)$

$\log_6 6 + \log_6 5 - \log_6 2$

$1 + y - x$

OR

$\log_6 (3 \cdot 5)$

$\log_6 3 + \log_6 5$

$1 - x + y$

4. Express y in terms of x if

$\ln y = \ln x - 2\ln 5$

$\ln y = \ln \frac{x}{5^2}$

$y = \frac{x}{25}$

5. Solve $\log_2 x + \log_2(x-2) = 3$

$\log_2[(x)(x-2)] = 3$

$\log_2(x^2 - 2x) = 3$

$2^3 = x^2 - 2x$

$8 = x^2 - 2x$

$0 = x^2 - 2x - 8$

$(x-4)(x+2)$

$x = 4, x \neq -2$ extraneous solution

A#45

~~A#2~~

Pg 200

2 – 34 even, 40 – 46 even, 51