5-4
The Number e and the Function e^x

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

$f(x) \to$ ____ as $x \to \infty$

$e = 2.71828$

Fill in the chart:

n	$\left(1+\frac{1}{n}\right)^n$
10	2.5937
100	2.7048
1000	2.7169
10000	2.7181
100000	

Applications in physics, statistics, calculus

Compound Interest:

Suppose you invest P dollars (the principal) at 12% annual interest.

- If interest is compounded semiannually, then 6% is added each 6 months. $P(1.06)^2$
- If interest is compounded quarterly, then 3% is added each 3 months, $P(1.03)^4$
- If interest is compounded monthly, then 1% is added each month $P(1.01)^{12}$

- $P(t) = P_0\left(1+\frac{r}{n}\right)^{nt}$

  compounded n times per year for t years
- $P(t) = Pe^{rt}$

  compounded continuously for t years

$$\left(1+\frac{1}{n}\right)^{n} = e$$

Example:

If you invest $15,000 at 8% annual interest rate, how much will you have at the end of 10 years if:

$P_0\left(1+\frac{r}{n}\right)^{nt}$

a) compounded quarterly? $n=4$ $t=10$, $r=.08$ $P_0=15{,}000$

$15000\left(1+\frac{.08}{4}\right)^{4\cdot 10} = \$33{,}120.60$

b) compounded monthly?

c) compounded continuously? $P_0 e^{rt}$

$15{,}000\, e^{.08(10)}$

$\$33{,}383.11$

d) What is the effective annual yield?

Effective annual yield: the actual percentage rate earned per year.

Example: Two hundred dollars deposited in a bank that compounds interest monthly yields $210.23 over 1 year. Find the effective annual yield.

$$210.23 = 200(1+r)^1$$

$$1.051 = 1+r$$

$$.051 = r$$

$$r = 5.1\%$$

Pg 189 2, 4, 5, 7, 10 – 12.

2a. Evaluate $\left(1-\frac{1}{n}\right)^n$ for n = 100, n = 10,000 and n = 1,000,000.

b. Compare your answers in part a) with an approximation for e^{-1}.

4. Evaluate: a. $e^{0.08}$ b. $e^{-0.08}$ c. $e^{4/3}$

5. Suppose you invest $1.00 at 6% annual interest. Calculate the amount that you would have alter one year if the interest is compounded

a) quarterly b) monthly c) continuously

7. One hundred dollars deposited in a bank that compounds interest quarterly yields $107.50 over 1 year. Find the effective annual yield.

10. With which plan would an investor earn more, Plan A or Plan B?

Plan A: An 8% annual rate compounded quarterly for 5 years.

Plan B: A 7.5% annual rate compounded daily for 5 years.