5-3
Exponential Functions

Exponential Function: $f(x) = ab^x$

$y = ab^x$

Example 1: Write the exponential function with $f(0) = 3$ and $f(3) = 648$.

$(0, 3)$, $(3, 648)$

$3 = ab^0$

$3 = a \cdot 1$

$a = 3$

$\frac{648}{3} = \frac{3b^3}{3}$

$\sqrt[3]{216} \quad \sqrt[3]{(b^3)}$

$6 = b$

$f(x) = 3 \cdot 6^x$

We often write exponential functions in the following form when they are used to describe growth or decay over time:

$A(t) = A_0(1 + r)^t$

A_0 = amt at time t = 0, r = growth rate

r > 0 growth r < 0 decay

OR: $A(t) = A_0 b^{t/k}$

k = time needed to multiply A_0 by b

Example 2: A bank advertises that if you open a savings account, you can double your money in 15 years. Express A(t), the amount of money after t years in one of the two forms listed above. Then find r, the annual growth rate of the account.

$A(t) = A_0 b^{t/k}$

$A(t) = A_0 (2)^{t/15}$

annual growth: $A_0\left[(2)^{1/15}\right]^t$

$A_0[1.047]^t$ $1 + r$

$r = 4.7\%$

The Rule Of 72: If a quantity is growing at r% per year (or month…) then the doubling time is approximately (72 ÷ r) years (or months…)

So, thinking about example 2,

15 years to double
find rate

$\frac{72}{r} = t$

$\frac{72}{r} = 15$

$\frac{72}{15} = \frac{15r}{15}$

$r = 4.8\%$

Example 3: A radioactive isotope has a ½ life of 5 days. At what rate does it decay each day?

$A(t) = A_0 b^{+1/k}$

$A(t) = A_0 \left(\frac{1}{2}\right)^{+1/5}$

$= A_0 \left(\left(\frac{1}{2}\right)^{1/5}\right)^t$

$= A_0 (.87)^t$

$1 - r$

$r = 13\%$

Pg 183 3, 6, 7, 9 – 16

Pg 173 9, 25, 35, 39, 44
Pg 178 10, 13, 15, 20, 22, 29, 37