

5-2  
Growth and Decay:  
Rational Exponents

$$(b^{1/2})^2 = b$$

$$(\sqrt{b})^2 = b$$

So...  $\sqrt{b} = b^{1/2}$

$$a^x = b^x$$


$$b^{3/2} = (b^{1/2})^3 \quad \text{or} \quad (b^3)^{1/2}$$
$$= (\sqrt{b})^3 \quad \sqrt{b^3}$$

SO... we define

$$b^{p/q} = \left(\sqrt[q]{b}\right)^p \quad \text{or} \quad \sqrt[q]{b^p}$$

Example 1.

Write in radical form:  $2^{4/5}$

$$\sqrt[5]{2^4}$$

$$\left(\sqrt[5]{2}\right)^4$$

Example 2.

Write in exponential form:  $\left(\sqrt[5]{10}\right)^2$

$$10^{2/5}$$

Example 3.

Evaluate without a calculator:  $8^{5/3}$

$$\sqrt[3]{8^5} = \left(\sqrt[3]{8}\right)^5$$

$$= (2)^5 = 32$$

Evaluate without a calculator:  $8^{-5/3}$

$$\frac{1}{32}$$

Example: Suppose a boat presently worth \$2200 depreciates at a rate of 15% a year. How much will it be worth in 2 years and 4 months from now?

$$\begin{aligned}
 A(t) &= A_0(1-r)^t \\
 &= 2200(1-.15)^{7/3} \\
 &= 2200(.85)^{7/3} \\
 &\approx \cancel{\$}1505.68
 \end{aligned}$$

$$\begin{aligned}
 2\frac{4}{12} \\
 2\frac{1}{3} &= \frac{7}{3}
 \end{aligned}$$

## Solving Exponential Equations:

Remember: if  $a^x = b^x$ , then  $a = b$ .

Remember:

If  $a^x = a^y$  then  $x = y$ .Example 5: Solve for  $x$ .  $3^x = 27$ 

$$3^x = 3^3$$

$$x = 3$$

Example 6: Solve for  $x$ .  $3^x = \left(\frac{1}{27}\right)^{x-1}$ 

$$3^x = (3^{-3})^{x-1}$$

$$3^x = 3^{-3x+3}$$

$$x = -3x + 3$$

$$4x = 3$$

$$x = \frac{3}{4}$$

A#39 pg 178 2, 4, 5, 8, 9, 14, 16, 17, 23, 26,  
28, 30, 33, 36, 38

Write each expression using a radical sign and no negative exponents.

2. a.  $3y^{2/5}$    b.  $(3y)^{2/5}$    c.  $a^{4/7}b^{-4/7}$    d.  $a^{1/10}b^{-1/5}$

Write each expression using positive rational exponents.

4. a.  $\sqrt[3]{8x^7}$    b.  $(\sqrt[4]{16x})^3$    c.  $\sqrt[3]{27x^{-6}y^2}$

d.  $\frac{\sqrt[4]{x} \sqrt[3]{x}}{\sqrt[6]{x}}$

Simplify:

5. a.  $(9/25)^{1/2}$    b.  $(9/25)^{-1/2}$

c.  $(9/25)^{5/2}$    d.  $(9/25)^{-1.5}$

8.  $(25^{-1/3})^{-3/2}$    9.  $(81^{1/2} - 9^{1/2})^2$    14.  $(4a^3)^{1/3} / (4a^3)^{-2/3}$

16. The value of a computer depreciates at the rate of 25% per year. If a computer is now worth \$2400, find its approximate value:

- a. 3 years 6 months from now  
b. 20 months ago

17.  $a^{1/2}(a^{3/2} - 2a^{1/2})$    23.  $\frac{2n^{1/3} - 4n^{-2/3}}{2n^{-2/3}}$

26.  $\frac{4ab^{-1/2} - 2ab^{1/2}}{(a^2b)^{-1/2}}$    28.  $\frac{\sqrt{(2x)^5}}{\sqrt{(2x)^9}}$

Solve:

30.  $9^{4x} = 81$    33.  $8^x = 2^7 \cdot 4^9$

36. a.  $(2x)^{-2} = 16$    b.  $2x^{-2} = 16$

c.  $4(x - 2)^{-2} = 16$

38. The price of firewood four years ago was \$140 per cord. Today a cord of wood costs \$182. To the nearest percent, what has been the annual rate of increase in the cost?

A#40

Pg 173 7, 11, 15, 17, 31, 37, 41

Pg 178 6, 7, 19, 27, 35