12.4 Sums of Infinite Geometric Series

Consider the infinite geometric series

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots$$

Let's look at the partial sums:

$S_1 = \frac{1}{2}$

$S_2 = \frac{1}{2}+\frac{1}{4} = \frac{3}{4}$

$S_3 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8} = \frac{7}{8}$

$S_4 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16} = \frac{15}{16}$

$S_5 = \frac{31}{32}$

What seems to be happening?

Let's use the formula for S_n. $S_n = \frac{t_1(1 - r^n)}{1 - r}$

What happens as $n \to \infty$?

The sum of an infinite geometric series:

$$S = \frac{a_1}{1 - r} \qquad |r| < 1$$

What if $|r| > 1$ or $= 1$?

$2+4+8+16+32+\ldots$

Examples:

1. Find the sum:

$$\frac{2}{5}+\frac{4}{25}+\frac{8}{125}+\frac{16}{625}+\cdots$$

$r=\frac{2}{5}$ $\quad S=\frac{a_1}{1-r}$

$$\frac{\frac{2}{5}}{1-\frac{2}{5}}=\frac{\frac{2}{5}}{\frac{3}{5}}=\frac{2}{5}\cdot\frac{5}{3}$$

$$\frac{2}{3}$$

2. Find the sum: $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}$ $\quad S=\frac{a_1}{1-r}$

$a_1=\left(\frac{1}{2}\right)^{1-1}=1$ $\quad \frac{1}{1-\frac{1}{2}}=2$

3. Find the sum: $\sum_{n=1}^{\infty} 3\left(\frac{5}{4}\right)^{n-1}$

$r = \frac{5}{4}$ $\left|\frac{5}{4}\right| < 1$ No!

NO sum

4. Find the sum: $3 - \frac{3}{4} + \frac{3}{16} - \frac{3}{64} + \ldots$

$r = -\frac{1}{4}$

$S = \frac{3}{1-\left(-\frac{1}{4}\right)}$ $\frac{3}{\frac{5}{4}} = \frac{12}{5}$

We can use infinite series to write repeating decimals as rational numbers.

5. Write 0.72727272… as a fraction in lowest terms.

$\frac{8}{11}$

$\frac{23}{99}$

2, 4, 6 - 8, 12, 14, 20, 21, for 25 and 26 be sure to show work as done in class, 37

2. ★ **WRITING** *Explain* how to tell whether the series $\sum_{i=1}^{\infty} a_1 r^{i-1}$ has a sum.

PARTIAL SUMS **For the given series, find and graph the partial sums S_n for $n = 1, 2, 3, 4$, and 5. *Describe* what happens to S_n as n increases.**

4. $\frac{2}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \cdots$

6. $\frac{1}{4} + \frac{5}{4} + \frac{25}{4} + \frac{125}{4} + \frac{625}{4} + \cdots$

FINDING SUMS **Find the sum of the infinite geometric series, if it exists.**

7. $\sum_{n=1}^{\infty} 8\left(\frac{1}{5}\right)^{n-1}$

8. $\sum_{k=1}^{\infty} -6\left(\frac{3}{2}\right)^{k-1}$

12. $\sum_{n=1}^{\infty} -5\left(\frac{2}{5}\right)^{n-1}$

14. $\sum_{n=1}^{\infty} \frac{1}{2}\left(-\frac{10}{3}\right)^{n-1}$

FINDING SUMS **Find the sum of the infinite geometric series, if it exists.**

20. $-\frac{1}{8} - \frac{1}{12} - \frac{1}{18} - \frac{1}{27} + \cdots$

21. $\frac{2}{3} - \frac{2}{9} + \frac{2}{27} - \frac{2}{81} + \cdots$

REWRITING DECIMALS **Write the repeating decimal as a fraction in lowest terms.**

25. 0.444. . .

26. 0.161616. . .

37. **TIRE SWING** A person is given one push on a tire swing and then allowed to swing freely. On the first swing, the person travels a distance of 14 feet. On each successive swing, the person travels 80% of the distance of the previous