

## 12.3 Analyze Geometric Sequences and Series

Geometric Sequence: a sequence in which the ratio of any term to the previous term is constant (called the common ratio,  $r$ )

(multiplying by the same # each time)

Examples: tell whether the sequence is geometric

a) 2, 4, 8, 16,

geometric

$$r = 2$$

$$\frac{4}{2} = 2 \quad \frac{8}{4} = 2 \quad \frac{16}{8} = 2$$

b) 10, 12, 16, ...

$$\frac{12}{10} = 1.2$$

$$\frac{16}{12} = 1.\bar{3}$$

NOT geometric

c) 625, 125, 25, 5, 1,

$$\frac{125}{625} = \frac{1}{5}$$

$$\frac{25}{125} = \frac{1}{5}$$

$$r = \frac{1}{5}$$

d) 1, -1, 1, -1, ..

$$\frac{-1}{1} = -1$$

$$\frac{1}{-1} = -1$$

$$r = -1$$

$$a_1$$

$$a_2 = a_1 r$$

$$a_3 = a_2 r = (a_1 r) r = a_1 r^2$$

$$a_4 = a_3 r = (a_1 r^2) r = a_1 r^3$$


$$a_n = a_1 r^{n-1}$$

Example:

Given the sequence 2, 4, 8, 16, 32, ...

write a rule for  $a_n$  and find  $a_{12}$

geometric  $r=2$

$$a_n = a_1 r^{n-1}$$

$$a_n = 2(2)^{n-1}$$

$$a_{12} = 2(2)^{12-1}$$

$$= 2(2)^{11}$$

$$4096$$

Example:

Give the sequence 3, -15, 75, -375, ...

write a rule for  $a_n$  and find  $a_8$

$$a_n = a_1 r^{n-1}$$

$$a_n = 3(-5)^{n-1}$$

$$a_8 = 3(-5)^{8-1}$$

$$-234,375$$

Example: If  $a_6 = -96$  and  $r = 2$ , write a rule for  $a_n$ .

$$a_6 = a_1 r^5$$

$$-96 = a_1 (2)^5$$

$$-3 = a_1$$

$$a_n = -3(2)^{n-1}$$

Example:

If  $a_2 = -12$  and  $a_4 = -3$ , write a rule.

# Geometric Series:

$$S_n = a_1 \left( \frac{1-r^n}{1-r} \right)$$

3,  $\frac{3}{2}$ ,  $\frac{3}{4}$ ,  $\frac{3}{8}$ ,  $\frac{3}{16}$   $a_1 = 6(-2)^1$

Example:  $\sum_{i=1}^5 3 \left( \frac{1}{2} \right)^{i-1}$

$$S_5 = 3 \left( \frac{1 - \frac{1}{2}^5}{1 - \frac{1}{2}} \right)$$

$$= 3 \left( \frac{\frac{31}{32}}{\frac{1}{2}} \right) = 3 \left( \frac{31}{16} \right) = \frac{93}{16}$$

Example:  $\sum_{i=1}^8 6(-2)^i$

$$S_8 = -12 \left( \frac{1 - (-2)^8}{1 - (-2)} \right)$$

$$= -12 \left( \frac{1 - 256}{3} \right)$$

$$= -12 \left( \frac{-255}{3} \right)$$

$$= +1020$$

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Pg 814 6, 8, 11, 15, 16, 19, 22, for 28,  
29, and 31 don't graph, 39, 43, 45, 48,  
49, 51, 58