10-3 Double-Angle and Half-Angle Formulas

$\sin 2\alpha$

Double Angle Formula

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha + \alpha) = \sin \alpha \cos \alpha + \cos \alpha \sin \alpha$$

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

$$\cos 2\alpha = \cos\alpha \cos\alpha - \sin\alpha \sin\alpha$$

$$\boxed{= \cos^2\alpha - \sin^2\alpha}$$

$$1 - \sin^2\alpha - \sin^2\alpha$$

$$\boxed{= 1 - 2\sin^2\alpha}$$

$$\cos^2\alpha - (1 - \cos^2\alpha)$$

$$\cos^2\alpha - 1 + \cos^2\alpha$$

$$\boxed{2\cos^2\alpha - 1}$$

Double Angle Formula

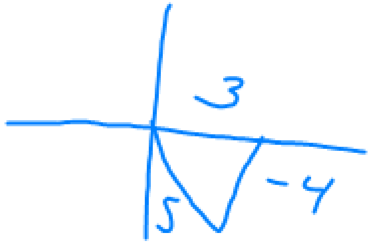
$$\sin 2\alpha = 2\sin\alpha \cos\alpha$$

$$\begin{aligned} \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= 1 - 2\sin^2\alpha \\ &= 2\cos^2\alpha - 1 \end{aligned}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \cancel{2}\tan^2\alpha}$$

Example: If $\sin\alpha = \dfrac{-4}{5}$, $270° \le \alpha \le 360°$,

find $\sin 2\alpha$, $\cos 2\alpha$, $\tan 2\alpha$



$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$= 1 - 2\left(-\frac{4}{5}\right)^2$$

$$1 - 2\left(\frac{16}{25}\right)$$

$$1 - \frac{32}{25}$$

$$\frac{-7}{25}$$

Example: Simplify: $\dfrac{2\tan 157.5°}{1 - \tan^2 157.5°}$ $= \tan(2 \cdot 157.5)$

$$\tan 315$$

$$-1$$

Half-Angle Formula

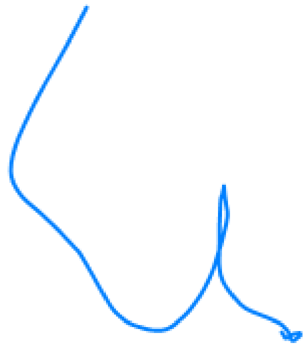
$$\sin \frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan \frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

$$= \frac{\sin \alpha}{1 + \cos \alpha}$$

$$= \frac{1 - \cos \alpha}{\sin \alpha}$$



Example: Evaluate $\sin(67.5^\circ)$

$$\sin\left(\frac{135^\circ}{2}\right) = \sqrt{\frac{1-\cos 135}{2}}$$

$$= \sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}}$$

$$\sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

Example: Evaluate $\cos\dfrac{\pi}{8}$

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Pg 383 1 - 25 odd