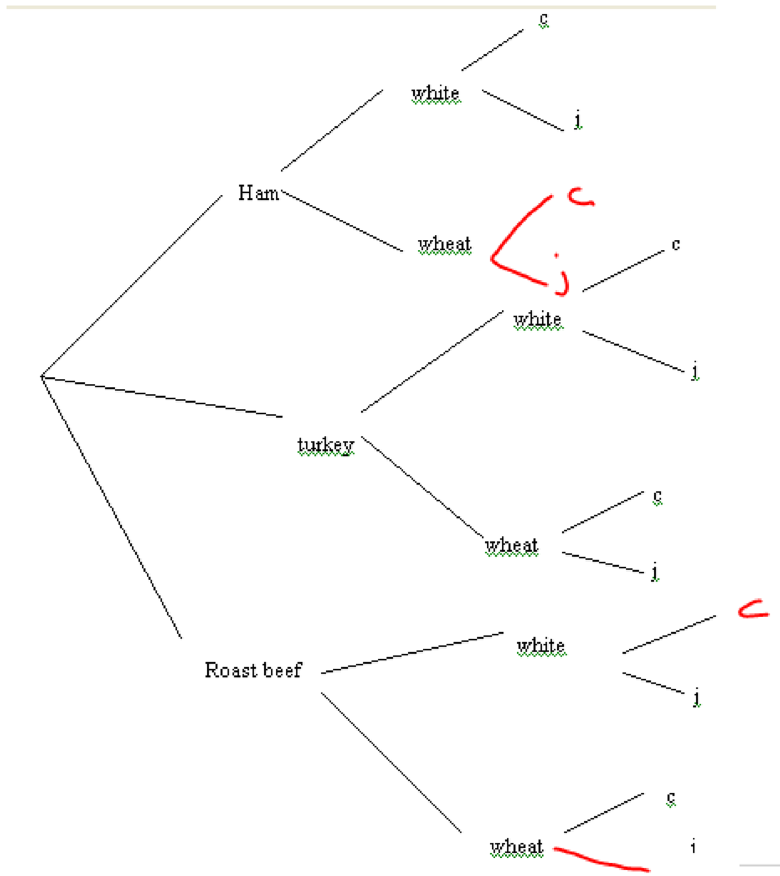
10.1 Fundamental Counting Principle

Example: You own a small deli. You offer 3 types of meat (ham, turkey, roast beef), 2 types of bread (white, wheat) and 2 types of cheese (cheddar, jack). How many sandwiches can you mak



12 sandwiches

$3 \cdot 2 \cdot 2$

<u>Fundamental Counting Principle</u>: If one event can occur in m ways, and another can occur in ~~m~~ n ways, then the number of ways that both can occur is $m \cdot n$

(this extends to any number of events).

Example: A new car has the following options:

Color	Engine Size	Transmission	Music
Red	4 cylinder	manual	radio 4 speakers
White	6 cylinder	automatic	radio 8 speakers
Blue			CD 4 speakers
			CD 8 speakers
			MP3 4 speakers
			MP3 8 speakers

$3 \cdot 2 \cdot 2 \cdot 6$

72

digit - 0 - 9
10

letter - 26

Example: Standard configuration for a New York license plate is 3 digits followed by 3 letters.

a) How many different license plates are possible if digits and letters can be repeated?

17,576,000

$$\underbrace{\underline{10} \cdot \underline{10} \cdot \underline{10}}_{\text{digits}} \cdot \underbrace{\underline{26} \cdot \underline{26} \cdot \underline{26}}_{\text{letters}}$$

b) How many different license plates are possible if digits and letters can not be repeated?

11,232,000

$$\underbrace{\underline{10} \cdot \underline{9} \cdot \underline{8}}_{\text{digits}} \cdot \underbrace{\underline{26} \cdot \underline{25} \cdot \underline{24}}_{\text{letters}}$$

Permutations: an arrangement of some or all of the elements of a set in definite order.

Example: I want to put 3 students, Alex, Brian, and Cindy into 3 seats. How many ways can I do this?

ABC
ACB
BAC
BCA
CAB
CBA

$\underline{3} \cdot \underline{2} \cdot \underline{1} = 6$

Factorial: $n! = n(n-1) \cdots 2 \cdot 1$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Sometimes you have more to choose from than you need.

Example: 12 skiers are competing.

A) In how many ways can they finish (assuming no ties)?

$12!$

$479{,}001{,}600$

B) In how many ways can 1^{st}, 2^{nd}, and 3^{rd} be awarded?

$$\underset{1^{st}}{\underline{12}} \cdot \underset{2^{nd}}{\underline{11}} \cdot \underset{3^{rd}}{\underline{10}}$$

1320

${}_nP_r$ = number of permutations of r objects from a group of n distinct objects.

$$= \frac{n!}{(n-r)!}$$

Example: In how many ways can a president and a vice president be chosen from a club of 15 people? $15 \cdot 14 = 210$ or ${}_{15}P_2$

To use your calculator:

${}_5P_3$ 60

5 4 3

${}_8P_4$ 1680

8 7 6 5

${}_7P_1$ 7

7

Permutations with repetition. E_1YE_2
If the E_1 and E_2 are distinct, there are 6
permutations.

3!

3 2 1

E_1YE_2
E_1E_2Y
YE_1E_2
YE_2E_1
E_2YE_1
E_2E_1Y

If E is just E and the two E's are
indistinguishable, then there are only 3
ways .

Permutations with Repetition: The number of distinguishable permutations of n objects where one object is repeated s_1 times, another s_2 times, etc, is

$$\frac{n!}{s_1! \cdot s_2! \cdot \ldots}$$

Example: How many distinguishable permutations are there of the word TALAHASSEE?

$$\frac{10!}{(3! \cdot 2! \cdot 2!)}$$

151,200

Pg 686 3, 6, 9 - 11, 14, 18, 23, 26, 30,
32, 36, 43, 46 - 48, 54, 62, 68, 70

Pg 561 3, 12, 14, 27, 30